Westlake University Dual-Listed Undergraduate and Graduate Courses Syllabus

1.Course Information

Course Name	Multiscale Modeling and Computation		Credits	3
Class Hours	Teaching Hours	Practice Hours	Lab Hours	Total
	48			48
Prerequisite courses (Consistent with major roadmap）				

2. Course Coordinator

Name	Zhennan Zhou	Contact Info	13717740158
Office Address	E4-238	Office Hours	Monday 9-11 am

3. Course Instructor

Name #1	Zhennan Zhou	Name #2	
Contact Info	Zhouzhennan@westlake.edu.cn	Contact Info	
Name #3		Name #4	
Contact Info		Contact Info	

4. Course Description（No more than 500 words）

This course explores the principles and computational methods essential for modeling multiscale systems prevalent in science and engineering, including materials science, fluid dynamics, and biological processes. Traditional single-scale models often fall short in capturing the complexity of these systems. This course introduces a hierarchy of physical models, from quantum mechanics and molecular dynamics to kinetic theory and continuum mechanics. It emphasizes analytical and computational techniques connecting microscopic mechanisms to macroscopic behavior, such as matched asymptotics, averaging, homogenization, multigrid methods, fast multipole methods, and Heterogeneous Multiscale Methods (HMM). Students will learn to build and analyze models based on first principles rather than empirical relations, tackling complex problems with multiscale coupling effects like equations with multiscale coefficients, dynamics across multiple timescales, and rare events. The course aims to cultivate a sophisticated multiscale perspective for modeling and computation, preparing students to address cutting-edge research and complex engineering challenges through a combination of theory, analysis, and computational practice.

5. Learning Objectives

Upon completing this course, students will be able to:

(1) Understand the hierarchy of physical models (e.g., quantum mechanics, molecular dynamics, kinetic theory, continuum mechanics) and their interrelations.
(2) Master key analytical methods for multiscale problems, including matched asymptotics, averaging, homogenization, and renormalization group methods.
(3) Apply classical and modern multiscale computational algorithms, such as multigrid, fast multipole methods, and Heterogeneous Multiscale Methods (HMM), to efficiently solve multiscale problems.
(4) Develop coupled models linking physical processes across different scales, moving beyond empirical constitutive laws.
(5) Analyze and solve complex problems exhibiting multiscale features, such as PDEs with multiscale coefficients, fast/slow dynamical systems, and rare event simulations.
(6) Develop a multiscale mindset for modeling and computation, enhancing their ability to tackle complex scientific and engineering problems.

6. Course Content

This course introduces the fundamental principles and computational techniques for analyzing and simulating systems involving multiple spatial and temporal scales. It covers the hierarchy of physical models from microscopic (quantum, atomistic) to macroscopic (kinetic theory, continuum mechanics) and explores the mathematical connections between them. Key analytical methods such as matched asymptotics, averaging, homogenization, and renormalization group ideas will be presented. The course also delves into computational strategies, including classical multiscale algorithms (multigrid, fast multipole methods, adaptive refinement, domain decomposition) and modern frameworks like the Heterogeneous Multiscale Method (HMM) and related concepts (e.g., equation-free, seamless coupling). Emphasis is placed on understanding how to capture macroscopic behavior efficiently, resolve localized phenomena (like defects or interfaces), and handle challenges like multiple time scales and rare events. Applications across various scientific and engineering disciplines will be used as illustrative examples.

7. Course Schedule

Week	**Session**	**Class Hour**	**Instructor(s)**	**Theme/Topic**	**Teaching activities（lecture/practical）**
1			Zhennan Zhou	Introduction to Multiscale Phenomena	Lecture
2			Zhennan Zhou	Hierarchy of Physical Models	Lecture
3			Zhennan Zhou	Analytical Methods I: Asymptotic Analysis	Lecture
4			Zhennan Zhou	Analytical Methods II: Averaging Methods	Lecture
5			Zhennan Zhou	Analytical Methods III: Homogenization	Lecture
6			Zhennan Zhou	Analytical Methods IV: Renormalization Group & Mori-Zwanzig	Lecture
7			Zhennan Zhou	Midterm Exam	Exam
8			Zhennan Zhou	Classical Numerical Methods	Lecture
9			Zhennan Zhou	Capturing Macroscale Behavior I: HMM Framework	Lecture
10			Zhennan Zhou	Capturing Macroscale Behavior II: HMM Examples	Lecture
11			Zhennan Zhou	Resolving Local Events	Lecture
12			Zhennan Zhou	Problems with Multiple Time Scales	Lecture
13			Zhennan Zhou	Extended Examples I	Lecture
14			Zhennan Zhou	Extended Examples II	Lecture
15			Zhennan Zhou	Students Presentations	Presentation
16			Zhennan Zhou	Students Presentations	Presentation

8. Assessment Weight

Type of Assessment	Percentage of Final Score	Notes
Attendance		
Class Performance		
Quiz		
Project	50	
Assignments	25	
Mid-term Exam	25	
Final Exam		
Other		

9. Grading

√ A. Graded

☐ B. Pass/Fail

☐ C. Hundred Point Scale

10. Textbook and Supplementary Readings
Textbook: Lecture notes.

Supplementary Readings:
Weinan E, Principles of Multiscale Modeling, Cambridge University Press, 2011.
Grigorios A. Pavliotis and Andrew M. Stuart, Multiscale Methods: Averaging and Homogenization (Texts in Applied Mathematics 53), Springer, 2008.